

Boundary Superstring Field Theory

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Using the Batalin-Vilkovisky formalism we provide a detailed analysis of the NS sector of boundary superstring field theory. We construct explicitly the relevant BV structure and derive the master action. Furthermore, we show that this action is exactly equal to the superdisk worldsheet partition function as was recently conjectured.

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1. INTRODUCTION

Recently there has been a resurgence of interest in the so-called boundary string field theory (BSFT) developed by Witten [1,2] and Shatashvili [3,4]. The main reason is that this version of open string field theory provides an intuitive framework for the study of tachyon condensation in open string theory [5,6,7]. In contrast to the cubic string field theory, where an infinite number of fields condense, BSFT allows for the tachyon field to condense alone, while the remaining open string tower continues to have vanishing expectation values [8].

Tachyon condensation of unstable D-brane systems is a completely classical phenomenon described in BSFT by the RG flow of the tachyon perturbation on the disk worldsheet. The exact expressions for the tachyon potential and the descent relations among D-branes, as expected from [9], can be derived in a straightforward manner in this approach.

The idea behind BSFT is to consider the classical open string field theory as a theory on the space of all boundary interactions on the disk with a fixed conformal worldsheet action in the bulk. The implementation of this idea relies on the use of the Batalin-Vilkovisky (BV) formalism [10,11]. This construction was initially presented in [1] for the bosonic string and provides a generic expression for the spacetime action that can be related to the disk partition function in a simple way ¹ [2,3,4]

$$S_B = Z - \beta^i \frac{\partial}{\partial \lambda^i} Z \quad (1)$$

with β^i the worldsheet beta function for the boundary coupling λ^i .

In the absence of an analogous construction for the superstring, the authors of [7] conjectured that the corresponding spacetime action S_F in the presence of worldsheet supersymmetry, should be exactly equal to the disk worldsheet partition function Z . This conjecture was motivated by a number of arguments involving the properties of Z at the conformal points, its finiteness, which is guaranteed by the presence of worldsheet supersymmetry, as well as similar proposals [12,13,14] ² in the context of a low-energy effective description. Further evidence for the validity of this choice was provided by the consistency of the results obtained from the analysis of tachyon condensation in superstring theory [7,16,17].

¹ To derive this relation one must also assume that ghosts and matter are decoupled.

² For a more recent discussion see also [15].

In this note we complete the above picture by repeating the analysis of [1] and applying the BV formalism on the space of supersymmetric worldsheet boundary perturbations on the disk. Our formulation of boundary superstring field theory (SBSFT) is cast entirely in superspace language. The crucial parts of the construction involve the definition of a fermionic vector field and the definition of the appropriate BV antibracket. The latter presents subtleties associated to the nature of the superconformal ghosts and the existence of different “pictures”. Nevertheless, an appropriate antibracket exists and the BV formalism provides a corresponding spacetime action, which turns out to be exactly equal to the disk partition function, as was conjectured in [7].

The organization of this paper is as follows. In section 2 we provide a brief review of the bosonic BSFT emphasizing the main points of the construction. In section 3 we analyze the boundary superstring field theory in detail. We construct the appropriate BV structure and use it to derive the equality $S_F = Z$. Moreover, since the spacetime action is a monotonically decreasing function along the RG flow, we may identify it with the boundary entropy on the disk. We present a further check of these points by performing explicit calculations in conformal perturbation theory up to third order. In section 4 we close with a short summary of our results and a brief discussion on related issues.

As this paper was being prepared for submission a preprint appeared [18] which addresses similar topics.

2. REVIEW OF BOSONIC BSFT

The worldsheet σ -model approach to string theory suggests that spacetime fields should be viewed as generalized coupling “constants” of two-dimensional worldsheet interactions. If we think of the string field as a collection of spacetime fields, this picture implies that the string field simply encodes the data of a two-dimensional field theory. Because of this, it is natural to expect that string field theory can be formulated on the “space of all two-dimensional field theories”.

Boundary string field theory is an open string field theory based on this idea. At the classical level its precise formulation relies on the application of the BV formalism³ on the configuration space of all two-dimensional field theories on the disk with arbitrary boundary interaction terms and a fixed conformal worldsheet action in the bulk.

³ The application of the BV formalism has also been very useful in the formulation of the open cubic string field theory [19,20] and closed string field theory [21].

In the following we review some basic features of the BV formalism and emphasize those that play a prominent role in the construction of BSFT. For a more detailed exposition on the BV formalism we refer the reader to [22,23].

One starts with a field configuration space \mathcal{M} equipped with a supermanifold structure, a non-degenerate closed odd two-form ω and a $U(1)$ symmetry under which ω has charge -1. We call this charge (BV) ghost number. The two-form ω gives rise to an antibracket, in the same way that a symplectic form gives rise to a Poisson bracket. In local coordinates u^I , the antibracket of two functions F and G is given by

$$\{F, G\} = \frac{\partial_r F}{\partial u^I} \omega^{IJ} \frac{\partial_l G}{\partial u^J}. \quad (2)$$

The focal point of the BV formalism is the *master equation*

$$\{S, S\} = 0 \quad (3)$$

for the master action S . This equation guarantees the gauge invariance of the system. Notice that because of the fermionic nature of ω the master equation is not trivially zero.

One way to satisfy the master equation identically is by choosing an appropriate fermionic vector field V on \mathcal{M} that has ghost number 1 and satisfies the equation

$$i_V \omega = dS. \quad (4)$$

Borrowing the language of the Hamiltonian formalism, we would say that V is the “Hamiltonian vector field” of the action S . As usual, $i_V \omega$ denotes the inner product of the vector V with the form ω and d is simply the exterior derivation on \mathcal{M} . The master equation will be satisfied identically if and only if V is nilpotent, i.e. $V^2 = 0$ and there exists at least one point where $V = 0$ [1]. The latter is a very natural requirement because the points of vanishing V are precisely the extrema of the spacetime action where the classical equations of motion are satisfied.

Moreover, the exterior derivative of equation (4) gives

$$d(i_V \omega) = 0. \quad (5)$$

Given the closedness of the antibracket, it is not hard to show that this is precisely the statement that ω is invariant under infinitesimal transformations generated by V , i.e. that

$$\mathcal{L}_V \omega = (di_V + i_V d)\omega = 0. \quad (6)$$

Hence, by the Poincare lemma, the property that the antibracket is V -invariant guarantees the existence of an action S given by (4). This is true, of course, only locally on the space of theories.

To summarize, the strategy for constructing a gauge invariant open string spacetime action involves two steps. First we define a nilpotent fermionic vector field V of ghost number 1 that plays the role of the “Hamiltonian” vector field of the action S and second we appropriately define a V -invariant odd symplectic two-form ω with ghost number -1. The master action S is then determined by (4).

This is the general setup. In the bosonic BSFT [1] the above construction works as follows. The configuration space \mathcal{M}_B consists, roughly speaking, of all possible two-dimensional worldsheet theories with a fixed conformal action $\mathcal{S}_{\text{bulk}}$ in the bulk of the disk (at the classical level) and a boundary action \mathcal{S}_{bdy} with arbitrary interactions on the boundary. After choosing, for instance, the standard flat action for the bulk, the dynamics of the two-dimensional field theories we want to consider are given by

$$\mathcal{S} = \mathcal{S}_{\text{bulk}} + \mathcal{S}_{\text{bdy}} = \frac{1}{4\pi\alpha'} \int_D d\sigma^1 d\sigma^2 \sqrt{h} (h^{ab} \partial_a X^\mu \partial_b X_\mu) + \frac{1}{2\pi} \int_D d\sigma^1 d\sigma^2 \sqrt{h} b^{ab} \partial_a c_b + \frac{1}{2\pi} \oint_{\partial D} d\tau \mathcal{V} \quad (7)$$

where D is the disk worldsheet, h^{ab} a rotationally invariant metric and τ a periodic coordinate on the boundary $\partial D = S^1$. Different points on the configuration space correspond to different choices of the boundary operator \mathcal{V} .

The vector field V is associated to the flow generated on \mathcal{M}_B by the bulk BRST charge Q . Under this definition it is straightforward to check that V has the required properties. It is nilpotent and has ghost number 1. Furthermore, when the classical equations of motion are satisfied, BRST invariance is restored and V is vanishing as expected.

The construction of the odd symplectic two-form ω is more involved. [1] first defines ω on-shell and later extends the definition off-shell. This extension, however, involves a subtle redefinition of the actual degrees of freedom of the theory and leads to an enlargement of the actual space of theories. This subtle point comes about in the following way. Tangent vectors to an on-shell submanifold of \mathcal{M}_B are spin one primary fields $\delta\mathcal{V}$. Since the BRST transformation of such fields should leave the worldsheet action invariant we deduce that there must be some operator O of ghost number 1, such that

$$\{Q, \delta\mathcal{V}\} = \partial_\tau O. \quad (8)$$

On-shell, O can be uniquely determined ⁴ from $\delta\mathcal{V}$. It satisfies the following two equations

$$\delta\mathcal{V} = b_{-1}O \quad (9)$$

and

$$\{Q, O\} = 0. \quad (10)$$

Off-shell, however, the second equation is no longer valid and the first one, which still makes sense, determines O only up to terms of the form $b_{-1}(\dots)$. Since O seems to be more fundamental than $\delta\mathcal{V}$ in some respects, it was proposed in [1] to consider an enlarged space of theories determined not only by the worldsheet action (7) but also by a ghost number 1 local operator O satisfying equation (9). Thus, it seems more appropriate to view \mathcal{M}_B as the space of the operators O and not as the space of the boundary perturbation operators \mathcal{V} .

Hence, given two vectors $\delta_1 O$ and $\delta_2 O$ at a point O of the enlarged configuration space, we define the odd symplectic form ω by

$$\omega(\delta_1 O, \delta_2 O) = (-)^{\epsilon(\delta_1 O)} \oint_{\partial D} d\tau_1 d\tau_2 \langle \delta_1 O(\tau_1) \delta_2 O(\tau_2) \rangle \quad (11)$$

with the correlation function being computed in the worldsheet theory with boundary interaction $\mathcal{V} = b_{-1}O$. This definition is slightly different from that of [1] by a sign factor. This factor is introduced in order to get the correct exchange property

$$\omega(\delta_1 O, \delta_2 O) = (-)^{(\epsilon_1+1)(\epsilon_2+1)+1} \omega(\delta_2 O, \delta_1 O) \quad (12)$$

where $\epsilon_i = \epsilon(\delta_i O)$. For a similar definition in the context of closed string field theory see for example [24]. Notice also that in the above expressions we still define the statistics of the arguments of ω as the natural statistics of the corresponding $\delta\mathcal{V}$ fields. For the precise off-shell definition of b_{-1} and a proof that (11) has the required properties, we refer the reader to [1].

Now that we have established the needed BV structure we can easily write down an expression for the master action S_B (up to an irrelevant sign) by using equation (4) and the explicit form of the fermionic vector field $V(O) = \{Q, O\}$

$$dS_B = \oint_{\partial D} d\tau_1 d\tau_2 \langle dO(\tau_1) \{Q, O\}(\tau_2) \rangle. \quad (13)$$

⁴ Up to a total derivative that has no effect on the worldsheet theory.

Under the simplifying assumption that ghosts and matter are decoupled one can set $O = c\mathcal{V}$. In that case, using different approaches, the authors of [2] and [3,4] proved that

$$S_B = Z - W^i \frac{\partial}{\partial \lambda^i} Z \quad (14)$$

where a generic expansion of the boundary operator $\mathcal{V} = \sum_i \lambda^i \mathcal{V}_i$ has been implied. W^i is a vector field on \mathcal{M}_B . More precisely, it was identified up to second order in conformal perturbation theory [3,4] with the beta function β^i , which corresponds to the worldsheet RG flow of the coupling λ^i .

3. BOUNDARY SUPERSTRING FIELD THEORY

a. The BV formalism of SBSFT

Boundary superstring field theory is formulated on the space \mathcal{M}_F of all worldsheet supersymmetric two-dimensional field theories on the superdisk with the usual NSR action in the bulk. In order to have a manifestly supersymmetric formalism we use a superspace notation. In particular, the worldsheet action takes the following form

$$\begin{aligned} \mathcal{S}_{NSR} &= \mathcal{S}_{\text{bulk}} + \mathcal{S}_{\text{bdy}} = \\ &= \frac{1}{4\pi\alpha'} \int d^2z d^2\theta D_{\bar{\theta}} \mathbf{X}^\mu D_\theta \mathbf{X}_\mu + \frac{1}{2\pi} \int d^2z d^2\theta B D_{\bar{\theta}} C + \int d\tau d\theta \mathcal{V}. \end{aligned} \quad (15)$$

Our conventions follow those of [25,26] and for the ghost and antighost superfields respectively we set

$$C(z, \theta) = c(z) + \theta\gamma(z) \quad (16)$$

$$B(z, \theta) = \beta(z) + \theta b(z). \quad (17)$$

The bottom and upper components of the superfield boundary perturbation will be generically given in the form

$$\mathcal{V}(\tau, \theta) = D(\tau) + \theta U(\tau). \quad (18)$$

For example, in the case of a single unstable non-BPS D9 brane in type IIA superstring theory, the tachyon perturbation is given by the worldsheet action

$$\mathcal{S}_{\text{bdy}} = \int_{\partial D} d\tau d\theta (\Gamma D\Gamma + \Gamma T(\mathbf{X})) \quad (19)$$

with $\Gamma = \eta + \theta F$ an auxiliary boundary fermion [7,8]. In that case

$$\mathcal{V} = \Gamma D\Gamma + \Gamma T(\mathbf{X}). \quad (20)$$

Furthermore, we can express the superconformal ghosts in a bosonized form [26]

$$\beta(z) = e^{-\phi(z)} \partial \xi(z), \quad \gamma(z) = e^{\phi(z)} \eta(z) \quad (21)$$

where ξ is a fermion of dimension 0 and η a fermion of dimension 1. Since we find this language very useful for the subsequent analysis, let us briefly recall a few relevant facts.

The zero-mode of ξ does not enter the above bosonized expressions and this results in a multiplicity of physically equivalent vacuum states that lead to different irreducible representations of the superconformal algebra, known as “pictures”. A vertex operator with a factor $e^{q\phi}$ is by definition in the q picture.

When calculating on-shell amplitudes, pictures can be used in a more or less arbitrary manner as long as the total superghost number of the insertions is -2. In terms of the bosonized form of the ghosts this condition implies a total ϕ -charge -2. For example, one may include two vertex operators in the -1 picture and the rest in the 0 picture [26]. Switching between different pictures can be achieved by the use of the picture changing operator

$$X = Q \cdot \xi = -\partial \xi c + e^{\phi} T_F^m - \partial \eta b e^{2\phi} - \partial(\eta b e^{2\phi}) \quad (22)$$

which *increases* the ϕ -charge by 1 or the use of the inverse picture changing operator [27,28]

$$Y = -\partial \xi c e^{-2\phi} \quad (23)$$

which *decreases* the ϕ -charge by 1.

The freedom of moving a picture changing operator inside an amplitude is not, however, a valid off-shell operation. This is an important subtlety of the superstring case and must be taken into account in the following manipulations.

The definition of V

In complete analogy to the bosonic case, it is again natural to associate the fermionic vector field V to the flow generated on \mathcal{M}_F by the bulk BRST charge Q . The only extra subtlety in the superstring case is that we choose \mathcal{M}_F as a space of superfields. Hence, a sensible definition of a vector field should generate flows that respect this property. More precisely, this property is satisfied if and only if the generator of the corresponding flow anticommutes with the generator of worldsheet SUSY. In our case, this is true since

$$\{Q, G_{-1/2}\} = 0. \quad (24)$$

Moreover, one can check that V also inherits the rest of the required properties. It is nilpotent, because Q is nilpotent, and has ghost number 1. We should emphasize that by ghost number we mean here the BV ghost number that coincides with the total ghost and superghost number of the bc and $\beta\gamma$ systems respectively. The details of this definition of V are the same as those of the bosonic case and we refer the reader to [1].

The definition of ω

We are looking for an appropriate two-form on the configuration space \mathcal{M}_F of supersymmetric boundary perturbations on the superdisk. In order to grow the right intuition about this form we first consider what happens on-shell. Let us also simplify the situation further by assuming that ghosts and matter are decoupled, so that the boundary interaction $\mathcal{V} = D + \theta U$ has no ghost dependence. In the NS sector the -1 picture vertex operator corresponding to \mathcal{V} will be of the form ⁵

$$\Lambda = -ce^{-\phi}D. \quad (25)$$

We obtain a 0 picture representation of this operator by acting with the picture changing operator X

$$X \cdot \Lambda = \gamma D - cU. \quad (26)$$

This expression is precisely the upper component of the superfield

$$G = C\mathcal{V} \quad (27)$$

⁵ The vertex operators given by Λ correspond to the so-called strongly physical states. For a related discussion see [28].

which is the natural supersymmetric generalization of the corresponding bosonic expression $O = c\mathcal{V}$. We propose that G should be considered as the fundamental object of boundary superstring field theory. This is actually analogous to the classical string field of the modified cubic superstring field theory [29,30,31] where the basic object is a 0 picture, ghost number 1 operator.

Given two superfield tangent vectors $\delta_1\mathcal{V}$ and $\delta_2\mathcal{V}$ we define the odd symplectic two-form ω in the superstring case as a two-point function at the perturbed point \mathcal{V} in the following way

$$\omega(\delta_1 G, \delta_2 G) = (-)^{\epsilon(\delta_1 G)} \int d\tau_1 d\tau_2 d\theta_1 d\theta_2 \langle Y(\tau_1) \delta_1 G(\tau_1, \theta_1) Y(\tau_2) \delta_2 G(\tau_2, \theta_2) \rangle \quad (28)$$

with $\delta_1 G$ and $\delta_2 G$ given by (27). The main difference with respect to the bosonic definition are the two insertions of the inverse picture changing operator Y in front of the 0 picture superfields. Their presence is required because a non-vanishing expectation value should have a total superghost number -2.

Expression (27), however, is only valid on-shell and only under the assumption of ghost-matter decoupling. Nevertheless, we can rewrite it as

$$\mathcal{V} = b_{-1} G. \quad (29)$$

As was explained in [1], this form also makes sense off-shell. Hence, for the off-shell definition of the antibracket, we propose to consider the defining relation (28) but with tangent vectors $\delta_1 G$ and $\delta_2 G$ given now (implicitly) by (29) and not by (27).

Equation (29) does not define the superfield G uniquely. As in the bosonic case, we circumvent this problem by considering an enlarged space of theories \mathcal{M}_F determined not only by the worldsheet action (15), but also by a 0 picture, ghost number 1 local superfield G satisfying (29). In some sense, we consider the superfields G as the fundamental degrees of freedom of the theory. Nevertheless, we still define the statistics of the arguments of ω as the natural statistics of the corresponding $\delta\mathcal{V}$ fields.

We also want to emphasize that contrary to our on-shell experience from the first quantized description of string theory, the position of the picture changing operators in the definition of ω is important. We cannot move them freely inside the correlator because this is not a valid off-shell operation.

The next step is to verify that ω given by (28) has all the required properties, i.e. that it is a V -invariant non-degenerate odd symplectic two-form with BV ghost number -1.

Since the total ghost number of the vacuum is -1 (-3 from the bc system and +2 from the $\beta\gamma$ system) and the ghost number of both insertions $Y \cdot \delta_i G$ is 0, we deduce that ω has BV ghost number -1, as expected.

The statistics $\epsilon(\omega)$, on the other hand, is given by the sum

$$\epsilon(\delta_1 G) + \epsilon(\delta_2 G) + 2\epsilon(Y) + 2 = \epsilon(\delta_1 G) + \epsilon(\delta_2 G) \pmod{2} \quad (30)$$

with the extra +2 in the left-hand side coming from the two θ integrations. Since a non-vanishing correlator requires $\epsilon(\delta_1 G) + \epsilon(\delta_2 G) = 1 \pmod{2}$ we get

$$\epsilon(\omega) = 1 \pmod{2} \quad (31)$$

and therefore ω is odd.

In order to show the non-degeneracy of the antibracket let us go on-shell. In that case ω vanishes for BRST exact insertions and therefore we may regard it as a two-form on the space of classical solutions. It is non-degenerate because it is related to the Zamolodchikov metric g on the space of conformal field theories. We can prove this by setting $\delta_1 G = C\mathcal{V}$ and $\delta_2 G = C\partial CW$, with V and W two spin 1/2 primary matter superfields. It follows that $\omega(\delta_1 G, \delta_2 G) \propto g(D_V, D_W)$, where D_V and D_W are the bottom components of the matter superfields. Thus, the non-degeneracy of ω follows from the non-degeneracy of the Zamolodchikov metric in complete analogy to the bosonic situation [1].

To prove that $d\omega = 0$, let us introduce local coordinates λ^i on \mathcal{M}_F . We can then write a generic superfield tangent vector δG in terms of the expansion $\delta G = \sum_i \lambda^i \delta_i G$. By definition, we have

$$d\omega(\delta_i G, \delta_j G, \delta_k G) = \frac{\partial}{\partial \lambda^i} \omega(\delta_j G, \delta_k G) \pm (\text{cyclic permutations}). \quad (32)$$

The derivative with respect to λ^i gives

$$\frac{\partial}{\partial \lambda^i} \omega(\delta_j G, \delta_k G) = \int \prod_{\beta=1}^3 d\tau_\beta d\theta_\beta (-)^{\epsilon_j} \left\langle \left(b_{-1} \delta_i G(\tau_1, \theta_1) \right) Y(\tau_2) \delta_j G(\tau_2, \theta_2) Y(\tau_3) \delta_k G(\tau_3, \theta_3) \right\rangle \quad (33)$$

with $\epsilon_i = \epsilon(\delta_i G)$. Hence, written explicitly, equation (32) takes the form

$$\begin{aligned} d\omega(\delta_i G, \delta_j G, \delta_k G) &= (-)^{\epsilon_j} \int \prod_{\beta=1}^3 d\tau_\beta d\theta_\beta \left(\left\langle (b_{-1} \delta_i G)(Y \delta_j G)(Y \delta_k G) \right\rangle \right. \\ &\quad \left. + (-1)^{\epsilon_i} \left\langle (Y \delta_i G)(b_{-1} \delta_j G)(Y \delta_k G) \right\rangle + (-1)^{\epsilon_i + \epsilon_j} \left\langle (Y \delta_i G)(Y \delta_j G)(b_{-1} \delta_k G) \right\rangle \right). \end{aligned} \quad (34)$$

The fact that the above expression vanishes follows from the Q invariance of the unperturbed correlators and the invariance under b_{-1} . The details of the relevant calculation can be found in appendix A.

The final property we have to check is V -invariance, i.e. $d(i_V\omega) = 0$. More explicitly, one must show that

$$\frac{\partial}{\partial \lambda^i} \int \prod_{\beta=1}^3 d\tau_\beta d\theta_\beta (-)^{\epsilon_j} \left\langle Y(\tau_1) \delta_j G(\tau_1, \theta_1) Y(\tau_2) [Q, G](\tau_2, \theta_2) \right\rangle \pm (i \leftrightarrow j) = 0. \quad (35)$$

For comments on this proof we refer the reader again to appendix A.

This concludes our discussion of the BV structure of the boundary superstring field theory. We now employ the above formalism to investigate the master action.

b. The relation between S_F and Z

The spacetime action of SBSFT (again up to an irrelevant sign factor) follows directly from equation (4), the definition of the vector field V and the definition of the odd symplectic form ω

$$dS_F = \int d\tau_1 d\tau_2 d\theta_1 d\theta_2 \langle Y(\tau_1) dG(\tau_1, \theta_1) Y(\tau_2) [Q, G](\tau_2, \theta_2) \rangle. \quad (36)$$

Furthermore, since we make the assumption that ghosts and matter are decoupled, we may set $G = C\mathcal{V}$, in which case the action takes the form

$$dS_F = \int d\tau_1 d\tau_2 d\theta_1 d\theta_2 \langle Y(\tau_1) C(\tau_1, \theta_1) d\mathcal{V}(\tau_1, \theta_1) Y(\tau_2) [Q, C\mathcal{V}](\tau_2, \theta_2) \rangle. \quad (37)$$

In the following discussion we perform an explicit calculation of this expression and demonstrate how it relates to the superdisk partition function Z .

Let us start by making the simplifying assumption that \mathcal{V} has a definite scaling dimension h . The commutator $[Q, C\mathcal{V}]$ is given more explicitly by

$$[Q, C\mathcal{V}] = \{Q, C\}\mathcal{V} - C\{Q, \mathcal{V}\}. \quad (38)$$

Moreover, the following two equations hold [26]

$$\{Q, C\} = C\partial_\tau C - \frac{1}{4}(D_\theta C)(D_\theta C) \quad (39)$$

$$\{Q, \mathcal{V}\} = C\partial_\tau \mathcal{V} - \frac{1}{2}(D_\theta C)(D_\theta \mathcal{V}) + h(\partial_\tau C)\mathcal{V}. \quad (40)$$

Substituting them back into equation (38) gives

$$[Q, C\mathcal{V}] = [(1-h)C\partial_\tau C - \frac{1}{4}(D_\theta C)(D_\theta C)]\mathcal{V} + \frac{1}{2}C(D_\theta C)(D_\theta \mathcal{V}). \quad (41)$$

An explicit calculation of the normal ordered expression $Y[Q, C\mathcal{V}]$ in components (see appendix B for details) gives

$$Y[Q, C\mathcal{V}] = \left((1-h)Yc\partial_\tau c - \frac{1}{4}Y\gamma^2 \right)\mathcal{V} + \frac{1}{2}\theta(Y\gamma^2 - Yc\partial_\tau c)D_\theta \mathcal{V} + \left(h - \frac{1}{2} \right)\theta c\partial_\tau c e^{-\phi} D. \quad (42)$$

Non-vanishing amplitudes require three insertions of the c ghost and therefore only the last term in the above expression contributes to the action. Hence,

$$dS_F = \left(\frac{1}{2} - h \right) \oint_{\partial D} d\tau_1 d\tau_2 \langle c(\tau_1)c(\tau_2)\partial_\tau c(\tau_2) \rangle_{bc} \langle e^{-\phi(\tau_1)}e^{-\phi(\tau_2)} \rangle_{\beta\gamma} \langle dD(\tau_1)D(\tau_2) \rangle_m. \quad (43)$$

Since

$$\langle c(\tau_1)c(\tau_2)\partial_\tau c(\tau_2) \rangle = 2(\cos(\tau_1 - \tau_2) - 1) = -4\sin^2\left(\frac{\tau_1 - \tau_2}{2}\right) \quad (44)$$

and

$$\langle e^{-\phi(\tau_1)}e^{-\phi(\tau_2)} \rangle = \frac{1}{2} \frac{1}{\sin\left(\frac{\tau_1 - \tau_2}{2}\right)} \quad (45)$$

we take

$$dS_F = (2h - 1) \oint_{\partial D} d\tau_1 d\tau_2 \sin\left(\frac{\tau_1 - \tau_2}{2}\right) \langle dD(\tau_1)D(\tau_2) \rangle. \quad (46)$$

For a generic perturbation \mathcal{V} parametrized by couplings λ^i and operators \mathcal{V}_i of conformal weight h_i

$$\mathcal{V} = \sum_i \lambda^i \mathcal{V}_i \quad (47)$$

and the above equation becomes

$$\frac{\partial S_F}{\partial \lambda^i} = \left(h_j - \frac{1}{2} \right) \lambda^j G_{ij}(\lambda) \quad (48)$$

where

$$G_{ij} = 2 \oint_{\partial D} d\tau_1 d\tau_2 \sin\left(\frac{\tau_1 - \tau_2}{2}\right) \langle D_i(\tau_1)D_j(\tau_2) \rangle. \quad (49)$$

These equations should be compared to equations (2.9) and (2.10) of reference [6] for the bosonic case.

For a unitary theory, the integral appearing in equation (49) is expected to give a positive definite metric G_{ij} . Indeed, the Hilbert space of a unitary theory should have,

by definition, a positive definite norm given by the time ordered expectation value. In particular, for a Hilbert space of odd excitations, this statement implies that we should be able to write an expectation value of the form $\langle D(x_1)D(x_2) \rangle$ as $\text{sign}(x_1 - x_2)f_D(x_1 - x_2)$, with f_D being a positive function. Combining this extra sign factor with the sine in the integral of (49) gives a manifestly positive form to the metric G_{ij} .

Also, equation (48) cannot be correct in general because $(\frac{1}{2} - h_j)\lambda^j$ is not a covariant expression on the space of theories. The relevant argument goes exactly the same way as in the bosonic case [6]. The correct covariant generalization of (48) is given by

$$\frac{\partial S_F}{\partial \lambda^i} = \beta^j G_{ij}. \quad (50)$$

Thus, along the RG flow

$$\frac{\partial S_F}{\partial \log t} = -\beta^i \frac{\partial S_F}{\partial \lambda_i} = -\beta^i \beta^j G_{ij} \quad (51)$$

where t is an RG (length) scale parameter. Since G_{ij} is positive definite, S_F is a monotonically decreasing function and also stationary at the conformal points. Furthermore, as we show in the following discussion, S_F equals to the disk partition function Z . Therefore, S_F can be identified with the boundary entropy and the above analysis agrees very nicely with the conjecture of [32,33] in the context of boundary CFT.

In order to prove the conjectured relation $S_F = Z$, we make use of the “two-systems” approach introduced in [2]. According to this, we assume that the matter system consists of two decoupled subsystems with partition functions Z_1 and Z_2 . Thus, the combined matter partition function Z equals the product $Z_1 Z_2$ and the expansion (47) takes the form

$$\mathcal{V} = \sum_i \lambda^i V_i + \sum_k \rho^k \tilde{V}_k \quad (52)$$

with λ^i couplings for the first system and ρ^k couplings for the second system. Accordingly, (46) becomes

$$dS_F = \oint_{\partial D} d\tau_1 d\tau_2 \sin\left(\frac{\tau_1 - \tau_2}{2}\right) \left\langle dD(\tau_1) \left(\sum_i (2h_i - 1) \lambda^i D_i(\tau_2) + \sum_k (2h_k - 1) \rho^k \tilde{D}_k(\tau_2) \right) \right\rangle. \quad (53)$$

After substituting into this equation the full expansion of dD we get two kinds of terms. One involves two point functions of the form $\langle D_i D_j \rangle$ and $\langle \tilde{D}_k \tilde{D}_l \rangle$. The other involves mixed terms of the form $\langle D_i \tilde{D}_k \rangle$. Since the two systems are decoupled these mixed correlators factorize into a product of two one-point functions, which are identically

zero because the bottom components D are fermionic. Moreover, even if these terms were not zero, the resulting expression would involve the integral of $\sin(\frac{\tau_1 - \tau_2}{2})$ over the circle and hence it would still be vanishing. The bosonic case, on the other hand, involved several non-vanishing mixed terms and these were responsible for the extra term with the β function on the right side of (14). Thus, we find that

$$dS_F = \oint_{\partial D} d\tau_1 d\tau_2 \sin\left(\frac{\tau_1 - \tau_2}{2}\right) \left(\sum_{i,j} (2h_i - 1) \lambda^i d\lambda^j \langle D_j(\tau_1) D_i(\tau_2) \rangle + \right. \\ \left. + \sum_{k,l} (2h_l - 1) \rho^l d\rho^k \langle \tilde{D}_k(\tau_1) \tilde{D}_l(\tau_2) \rangle \right). \quad (54)$$

After setting

$$a = \sum_j a_j(\lambda) d\lambda^j = \sum_j d\lambda^j \left[\oint_{\partial D} d\tau_1 d\tau_2 \sin\left(\frac{\tau_1 - \tau_2}{2}\right) \sum_i (2h_i - 1) \lambda^i \langle D_j(\tau_1) D_i(\tau_2) \rangle_1 \right] \quad (55)$$

$$\tilde{a} = \sum_k \tilde{a}_k(\rho) d\rho^k = \sum_k d\rho^k \left[\oint_{\partial D} d\tau_1 d\tau_2 \sin\left(\frac{\tau_1 - \tau_2}{2}\right) \sum_l (2h_l - 1) \rho^l \langle \tilde{D}_k(\tau_1) \tilde{D}_l(\tau_2) \rangle_2 \right] \quad (56)$$

we can write equation (54) in a compact form as

$$dS_F = aZ_2 + Z_1\tilde{a}. \quad (57)$$

The one-forms a and \tilde{a} can be related to the partition functions Z_1 and Z_2 by using the fact that $d^2 S_F = 0$. Setting to zero the coefficient of $d\lambda^i \wedge d\rho^k$ in the expression of $d^2 S_F$ gives

$$dZ_1 \tilde{a} - a dZ_2 = 0. \quad (58)$$

Since the two systems are decoupled, this implies the existence of a non-zero constant g such that

$$a = g dZ_1, \quad \tilde{a} = g dZ_2. \quad (59)$$

g cannot be zero because ω is non-degenerate. In fact, as we show in the next section, conformal perturbation theory fixes the value of g to 1. Thus, putting everything together gives

$$dS_F = d(Z_1 Z_2) \quad (60)$$

or

$$S_F = Z. \quad (61)$$

The above derivation of the equality between the master action and the superdisk partition function was carried under the assumption that the matter system consists of two decoupled subsystems. However, as pointed out in [2], this restriction is not necessary. One can consider any matter system and carry out the above analysis by adding an auxiliary decoupled system. At the end of the calculation the auxiliary system can be suppressed by setting its couplings to a fixed value.

c. Some calculations in conformal perturbation theory

In the previous section we used a general argument to show the relation

$$\partial_i S_F = g \partial_i Z \quad (62)$$

where g is a constant factor. We examine now this relation from the point of view of conformal perturbation theory up to 3rd order on the bare couplings λ^i of the boundary perturbations. In particular, we consider the following expansion of equations (48) and (49) around the conformal point

$$\begin{aligned} \partial_i S_F = 2(h_j - \frac{1}{2}) & \left(\lambda^j \oint d\tau_1 d\tau_2 \sin \frac{\tau_1 - \tau_2}{2} \langle D_i(\tau_1) D_j(\tau_2) \rangle_0 + \right. \\ & \left. \lambda^j \lambda^k \oint d\tau_1 d\tau_2 d\tau_3 \sin \frac{\tau_1 - \tau_2}{2} \langle D_i(\tau_1) D_j(\tau_2) U_k(\tau_3) \rangle_0 \right) \end{aligned} \quad (63)$$

and compare it to the corresponding expansion of the disk partition function

$$\partial_i Z = \lambda^j \oint d\tau_1 d\tau_2 \langle U_i(\tau_1) U_j(\tau_2) \rangle_0 + \frac{1}{2} \lambda^j \lambda^k \oint d\tau_1 d\tau_2 d\tau_3 \langle U_i(\tau_1) U_j(\tau_2) U_k(\tau_3) \rangle_0. \quad (64)$$

According to (62), we should be able to verify the following two equations

$$2(h_j - \frac{1}{2}) \oint d\tau_1 d\tau_2 \sin \frac{\tau_1 - \tau_2}{2} \langle D_i(\tau_1) D_j(\tau_2) \rangle_0 = g \oint d\tau_1 d\tau_2 \langle U_i(\tau_1) U_j(\tau_2) \rangle_0 \quad (65)$$

in second order, and

$$\begin{aligned} 2(h_j - \frac{1}{2}) \oint d\tau_1 d\tau_2 d\tau_3 \sin \frac{\tau_1 - \tau_2}{2} \langle D_i(\tau_1) D_j(\tau_2) U_k(\tau_3) \rangle_0 + (j \leftrightarrow k) = \\ g \oint d\tau_1 d\tau_2 d\tau_3 \langle U_i(\tau_1) U_j(\tau_2) U_k(\tau_3) \rangle_0. \end{aligned} \quad (66)$$

in third order.

For the second order computation, the explicit form of the correlators is as follows

$$\langle D_i(\tau_1)D_j(\tau_2)\rangle_0 = \frac{c\delta_{h_i,h_j}}{\left|\sin \frac{\tau_1-\tau_2}{2}\right|^{2h}} \text{sign}(\tau_1 - \tau_2), \quad (67)$$

$$\langle U_i(\tau_1)U_j(\tau_2)\rangle_0 = \frac{b\delta_{h_i,h_j}}{\left|\sin \frac{\tau_1-\tau_2}{2}\right|^{2h+1}}, \quad (68)$$

with $h = h_i = h_j$. After evaluating the relevant integrals by using the general expression

$$\int_0^{2\pi} \frac{d\tau_1}{2\pi} \frac{d\tau_2}{2\pi} \left|\sin \frac{\tau_1 - \tau_2}{2}\right|^z = \frac{1}{\sqrt{\pi}} \frac{\Gamma(\frac{1}{2} + \frac{z}{2})}{\Gamma(1 + \frac{z}{2})}, \quad (69)$$

equation (65) becomes

$$2(h - \frac{1}{2})c \frac{\Gamma(1-h)}{\Gamma(\frac{3}{2}-h)} = gb \frac{\Gamma(-h)}{\Gamma(\frac{1}{2}-h)} \Rightarrow \quad (70)$$

$$2ch = gb.$$

This equation, with $g = 1$, is a consequence of the SUSY Ward identities on the worldsheet.

For a short derivation see appendix C.

Similarly, in third order we have the correlators

$$\langle D_i(\tau_1)D_j(\tau_2)U_k(\tau_3)\rangle_0 = \frac{C_{ijk} \text{sign}(\tau_1 - \tau_2)}{\left|\sin \frac{\tau_1-\tau_2}{2}\right|^{h_i+h_j-h_k-\frac{1}{2}} \left|\sin \frac{\tau_2-\tau_3}{2}\right|^{h_j+h_k-h_i+\frac{1}{2}} \left|\sin \frac{\tau_1-\tau_3}{2}\right|^{h_i+h_k-h_j+\frac{1}{2}}}, \quad (71)$$

$$\langle U_i(\tau_1)U_j(\tau_2)U_k(\tau_3)\rangle_0 = \frac{B_{ijk}}{\left|\sin \frac{\tau_1-\tau_2}{2}\right|^{h_i+h_j-h_k+\frac{1}{2}} \left|\sin \frac{\tau_2-\tau_3}{2}\right|^{h_k+h_j-h_i+\frac{1}{2}} \left|\sin \frac{\tau_1-\tau_3}{2}\right|^{h_i+h_k-h_j+\frac{1}{2}}}. \quad (72)$$

By using the general expression

$$\int_0^{2\pi} \frac{d\tau_1}{2\pi} \frac{d\tau_2}{2\pi} \frac{d\tau_3}{2\pi} \left|\sin \frac{\tau_1 - \tau_2}{2}\right|^a \left|\sin \frac{\tau_2 - \tau_3}{2}\right|^b \left|\sin \frac{\tau_1 - \tau_3}{2}\right|^c = \frac{1}{\pi^{3/2}} \frac{\Gamma(\frac{1}{2}(1+a))\Gamma(\frac{1}{2}(1+b))\Gamma(\frac{1}{2}(1+c))\Gamma(1+\frac{1}{2}(a+b+c))}{\Gamma(1+\frac{1}{2}(a+b))\Gamma(1+\frac{1}{2}(b+c))\Gamma(1+\frac{1}{2}(a+c))}. \quad (73)$$

we can evaluate all terms in (66). The result of this calculation for $g = 1$ is

$$-2B_{ijk}\left(\frac{1}{2}-h_i\right) = \left(\frac{1}{2}-h_i-h_j-h_k\right)\left(C_{ijk}\left(\frac{1}{2}+h_k-h_i-h_j\right)+C_{ikj}\left(\frac{1}{2}+h_j-h_i-h_k\right)\right). \quad (74)$$

Since $C_{ijk} = C_{ikj}$, the above equation becomes

$$B_{ijk} = -\left(\frac{1}{2} - h_i - h_j - h_k\right)C_{ijk}. \quad (75)$$

Again, we can verify this relation, as well as the symmetry properties of the constants C_{ijk} , by using the SUSY Ward identities. The relevant details can be found in appendix C.

Another important statement of the previous section was the identification of the worldsheet partition function Z with the boundary entropy of [32,33]. We can check this identification by an explicit conformal perturbation theory calculation, showing the decrease of the partition function under the renormalization group flow. We perturb the worldsheet action by some relevant operator very close to marginality and as a result the worldsheet theory flows towards a nearby conformal fixed point, where we can still use perturbation theory to calculate the new value of the partition function. A similar calculation can be found in appendix E of reference [33].

Thus, consider the boundary perturbation \mathcal{V} with conformal weight $h = 1/2 - y$ and $0 < y \ll 1/2$. For simplicity, we assume that the RG flow is closed under this perturbation and that there is no mixing with other fields. The corresponding beta function up to second order equals

$$\beta(\lambda) = -y\lambda - \frac{B}{\pi}\lambda^2, \quad (76)$$

with $B = B_{\mathcal{V}\mathcal{V}\mathcal{V}}$ the 3-point function constant appearing in (72). This β function implies the existence of a nearby fixed point given by

$$\beta(\lambda^*) = 0 \Rightarrow \lambda^* = -\frac{y\pi}{B} \ll 1. \quad (77)$$

In order to verify (76), consider an ultraviolet (length scale) cut-off l , an RG length scale parameter t and the boundary perturbation for bare coupling λ written in the form

$$S_{\text{bdy}} = \left(\frac{t}{l}\right)^y \lambda \oint \frac{d\tau}{2\pi} U(\tau). \quad (78)$$

In this relation, λ , τ and U are dimensionless.

Expanding the partition function to quadratic order in the coupling and using the OPE

$$U(\tau_1)U(\tau_2) \sim \frac{B}{|\sin \frac{\tau_1 - \tau_2}{2}|^{h+1/2}} U(\tau_2) \quad (79)$$

gives

$$\begin{aligned}
e^{S_{\text{bdy}}} &= 1 + \left(\frac{t}{l}\right)^y \lambda \oint \frac{d\tau}{2\pi} U(\tau) + \frac{1}{2} \left(\frac{t}{l}\right)^{2y} \lambda^2 \oint \frac{d\tau_1}{2\pi} \frac{d\tau_2}{2\pi} U(\tau_1) U(\tau_2) \\
&= 1 + \left(\frac{t}{l}\right)^y \lambda \oint \frac{d\tau}{2\pi} U(\tau) + \frac{1}{2} \left(\frac{t}{l}\right)^{2y} \lambda^2 \oint \frac{d\tau_1}{2\pi} U(\tau_1) \oint \frac{d\tau_2}{2\pi} \frac{B}{|\sin \frac{\tau_1 - \tau_2}{2}|^{h+1/2}} \\
&= 1 + \left(\frac{t}{l}\right)^y \lambda \oint \frac{d\tau}{2\pi} U(\tau) + \frac{1}{2} \left(\frac{t}{l}\right)^{2y} \lambda^2 \frac{B}{\sqrt{\pi}} \frac{\Gamma(\frac{y}{2})}{\Gamma(\frac{1+y}{2})} \oint \frac{d\tau}{2\pi} U(\tau) \\
&\sim 1 + \left(\frac{t}{l}\right)^y \lambda \oint \frac{d\tau}{2\pi} U(\tau) + \left(\frac{t}{l}\right)^{2y} \lambda^2 \frac{B}{y\pi} \oint \frac{d\tau}{2\pi} U(\tau).
\end{aligned} \tag{80}$$

In the last step we have set $y \sim 0$. Considering now these terms as a correction to the initial perturbation, we find

$$\frac{\delta\lambda(t)}{\delta \ln t} = y\lambda(t) + \frac{B}{\pi} \lambda(t)^2, \tag{81}$$

which gives precisely the beta function (76).

In terms of the bare coupling $\lambda = \lambda(l)$, the partition function up to 3rd order is given by

$$Z = 1 + \frac{1}{2} \left(\frac{t}{l}\right)^{2y} \lambda^2 \oint \frac{d\tau_1}{2\pi} \frac{d\tau_2}{2\pi} \langle U(\tau_1) U(\tau_2) \rangle_0 + \frac{1}{6} \left(\frac{t}{l}\right)^{3y} \lambda^3 \oint \frac{d\tau_1}{2\pi} \frac{d\tau_2}{2\pi} \frac{d\tau_3}{2\pi} \langle U(\tau_1) U(\tau_2) U(\tau_3) \rangle_0. \tag{82}$$

After substituting the CFT expressions (68) (with normalization $b = 1$) and (72) we obtain

$$Z = 1 + \frac{1}{2} \left(\frac{t}{l}\right)^{2y} \lambda^2 \frac{1}{\sqrt{\pi}} \frac{\Gamma(y - \frac{1}{2})}{\Gamma(y)} + \frac{1}{6} \left(\frac{t}{l}\right)^{3y} \lambda^3 B \frac{1}{\pi\sqrt{\pi}} \frac{(\Gamma(\frac{1}{2}y))^3 \Gamma(-\frac{1}{2} + \frac{3}{2}y)}{(\Gamma(y))^3}, \tag{83}$$

which in the $y \sim 0$ limit simplifies to

$$Z = 1 - y\lambda^2 \left(\frac{t}{l}\right)^{2y} - \frac{8}{3\pi} B \lambda^3 \left(\frac{t}{l}\right)^{3y}. \tag{84}$$

This equation can be re-expressed in terms of the renormalized coupling $\lambda(t)$ by solving the β function equation (81). The solution gives

$$\lambda = \left(\frac{t}{l}\right)^{-y} \frac{\lambda(t)}{1 - \frac{\lambda(t)}{\lambda^*} (1 - (\frac{t}{l})^{-y})}. \tag{85}$$

Expanding this expression up to second order in $\lambda(t)$ and substituting the result into (84) gives

$$Z = 1 - y\lambda(t)^2 - \frac{2}{3\pi} \lambda(t)^3 B. \tag{86}$$

Thus, in the IR limit where $\lambda(t) \rightarrow \lambda^*$, the total change of the disk partition function between the UV and IR fixed points becomes

$$\delta Z = -\frac{\pi^2 y^3}{3B^2}. \quad (87)$$

This result was also obtained in [33]. As expected, we find that Z decreases under the renormalization group flow.

5. DISCUSSION

The aim of the above discussion was to show that on a formal level the construction of the (classical) boundary string field theory for the NS sector of the superstring is more or less parallel to the analogous construction of the bosonic case. However, in the superstring case there are certain extra subtle points. The first is the requirement to preserve worldsheet supersymmetry. We satisfy it by using a superspace formalism that is manifestly supersymmetric. A second subtlety in the superstring case arises from the involved nature of the superconformal ghosts. Vertex operators can be chosen in different pictures and this choice involves the insertion of appropriate picture changing operators inside the correlation functions.

After defining the appropriate BV structure the spacetime action is determined by (36). Under the additional assumption of ghost and matter decoupling, this action takes a simpler and more appropriate for calculations form. Using the two-systems analysis of [2] and conformal perturbation theory up to third order we verified that the master action is non other than the disk partition function, exactly as it was conjectured in [7]. This identification also suggests that the spacetime action can also be thought of as the boundary entropy of [32,33]. It takes the right value at the conformal points and decreases along the RG flow.

Nevertheless, this construction of boundary superstring field theory is not at all complete. First of all, the above analysis has to be extended appropriately to include the R sector. Secondly, the presented formalism is plagued by the same problems that characterize the analogous construction of the bosonic case. Very simply put, the construction is too formal. The space of all theories (with varying local boundary interaction terms) gives rise to serious ultraviolet divergences, especially when one tries to add non-renormalizable boundary interaction terms. These terms correspond to higher massive excitations of the

open string and they certainly have to be included in any acceptable formulation of string field theory.

In order to tackle these divergences an appropriate cut-off has to be introduced. The cut-off should respect the rotational invariance, the b_{-1} Ward identities and V -invariance (i.e. the invariance under the closed BRST charge Q)⁶. In the superstring case it should also respect worldsheet supersymmetry. A cut-off can be chosen to respect all of the above symmetries except for V -invariance. Because of this, at the end of the calculation one would like to remove the cut-off in such a way that the V -invariance of the antibracket is restored. The relevant discussion of [34] for certain integrable boundary interactions in the bosonic case revealed that the removal of the cut-off presented difficulties. It is not clear however whether this poses an insurmountable obstacle in making sense of the notion of a space of open string theories with local boundary interactions. One expects similar difficulties in the superstring case as well.

Because of such problems the question of whether this formalism can provide a rigorous and full formulation of open string field theory is still open. It might be possible, however, that a more careful application of the BV formalism could provide the needed resolution of the above subtleties.

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⁶ Given the previous invariances, V -invariance is equivalent to the statement that the boundary interaction does not modify the BRST charge. See [4,34] for further comments on this point.

Appendix A

By definition we have

$$d\omega(\delta_i G, \delta_j G, \delta_k G) = (-)^{\epsilon_j} \int \prod_{\beta=1}^3 d\tau_\beta d\theta_\beta \left(\left\langle (b_{-1} \delta_i G)(Y \delta_j G)(Y \delta_k G) \right\rangle \right. \\ \left. + (-1)^{\epsilon_i} \left\langle (Y \delta_i G)(b_{-1} \delta_j G)(Y \delta_k G) \right\rangle + (-1)^{\epsilon_i + \epsilon_j} \left\langle (Y \delta_i G)(Y \delta_j G)(b_{-1} \delta_k G) \right\rangle \right), \quad (88)$$

with $\epsilon_i = \epsilon(\delta_i G)$.

Let us consider the following Ward identities for b_{-1}

$$\langle b_{-1}(\delta_i G(Y \delta_j G)(Y \delta_k G)) \rangle = 0 \Rightarrow \\ \langle (b_{-1} \delta_i G)(Y \delta_j G)(Y \delta_k G) \rangle + (-)^{\epsilon_i} \langle \delta_i G(b_{-1} Y \delta_j G)(Y \delta_k G) \rangle + \\ (-)^{\epsilon_i + \epsilon_j} \langle \delta_i G(Y \delta_j G)(b_{-1} Y \delta_k G) \rangle = 0,$$

$$\langle b_{-1}((Y \delta_i G) \delta_j G(Y \delta_k G)) \rangle = 0 \Rightarrow \\ \langle (b_{-1} Y \delta_i G) \delta_j G(Y \delta_k G) \rangle + (-)^{\epsilon_i} \langle (Y \delta_i G)(b_{-1} \delta_j G)(Y \delta_k G) \rangle + \\ (-)^{\epsilon_i + \epsilon_j} \langle (Y \delta_i G) \delta_j G(b_{-1} Y \delta_k G) \rangle = 0,$$

$$\langle b_{-1}((Y \delta_i G)(Y \delta_j G) \delta_k G) \rangle = 0 \Rightarrow \\ \langle (b_{-1} Y \delta_i G)(Y \delta_j G) \delta_k G \rangle + (-)^{\epsilon_i} \langle (Y \delta_i G)(b_{-1} Y \delta_j G) \delta_k G \rangle + \\ (-)^{\epsilon_i + \epsilon_j} \langle (Y \delta_i G)(Y \delta_j G)(b_{-1} \delta_k G) \rangle = 0.$$

Adding the last three identities, separating the appropriate terms and integrating gives

$$d\omega(\delta_i G, \delta_j G, \delta_k G) = \\ (-)^{\epsilon_j + 1} \int \prod_{\beta=1}^3 d\tau_\beta d\theta_\beta \left((-)^{\epsilon_i} \langle \delta_i G(b_{-1} Y \delta_j G)(Y \delta_k G) \rangle + (-)^{\epsilon_i + \epsilon_j} \langle \delta_i G(Y \delta_j G)(b_{-1} Y \delta_k G) \rangle + \right. \\ \langle (b_{-1} Y \delta_i G) \delta_j G(Y \delta_k G) \rangle + (-)^{\epsilon_i + \epsilon_j} \langle (Y \delta_i G) \delta_j G(b_{-1} Y \delta_k G) \rangle + \\ \left. \langle (b_{-1} Y \delta_i G)(Y \delta_j G) \delta_k G \rangle + (-)^{\epsilon_i} \langle (Y \delta_i G)(b_{-1} Y \delta_j G) \delta_k G \rangle \right).$$

We have three pairs of terms, each of them labelled by the same statistics factor in front. These pairs are actually vanishing. To see that, let us consider for example the pair

$$\langle (b_{-1} Y \delta_i G) \delta_j G(Y \delta_k G) \rangle + \langle (b_{-1} Y \delta_i G)(Y \delta_j G) \delta_k G \rangle. \quad (89)$$

For the unperturbed correlator $\langle \dots \rangle_0$ we can write the following two Ward identities for the BRST charge Q :

$$\begin{aligned}
& \left\langle \xi(0) Q \left((b_{-1} Y \delta_i G)(\tau_1, \theta_1) (\xi Y \delta_j G)(\tau_2, \theta_2) (Y \delta_k G)(\tau_3, \theta_3) e^{\int d\tau d\theta \mathcal{V}} \right) \right\rangle_0 = \\
& \left\langle X(0) (b_{-1} Y \delta_i G)(\tau_1, \theta_1) (\xi Y \delta_j G)(\tau_2, \theta_2) (Y \delta_k G)(\tau_3, \theta_3) \right\rangle \Rightarrow \\
& \langle \xi(0) (Q b_{-1} Y \delta_i G)(\xi Y \delta_j G)(Y \delta_k G) \rangle + \\
& (-)^{1+\epsilon_i} \langle \xi(0) (b_{-1} Y \delta_i G) \delta_j G (Y \delta_k G) \rangle + (-)^{\epsilon_i} \langle \xi(0) (b_{-1} Y \delta_i G) (\xi (Q Y \delta_j G)) (Y \delta_k G) \rangle + \\
& (-)^{\epsilon_i + \epsilon_j} \langle \xi(0) (b_{-1} Y \delta_i G) (\xi Y \delta_j G) (Q Y \delta_k G) \rangle + \\
& (-)^{\epsilon_i + \epsilon_j + \epsilon_k} \langle \xi(0) (b_{-1} Y \delta_i G) (\xi Y \delta_j G) (Y \delta_k G) [Q, e^{\int d\tau d\theta \mathcal{V}}] \rangle_0 = \\
& \langle X(0) (b_{-1} Y \delta_i G) (\xi Y \delta_j G) (Y \delta_k G) \rangle
\end{aligned}$$

and

$$\begin{aligned}
& \left\langle \xi(0) Q \left((b_{-1} Y \delta_i G)(\tau_1, \theta_1) (Y \delta_j G)(\tau_2, \theta_2) (\xi Y \delta_k G)(\tau_3, \theta_3) e^{\int d\tau d\theta \mathcal{V}} \right) \right\rangle_0 = \\
& \left\langle X(0) (b_{-1} Y \delta_i G)(\tau_1, \theta_1) (Y \delta_j G)(\tau_2, \theta_2) (\xi Y \delta_k G)(\tau_3, \theta_3) \right\rangle \Rightarrow \\
& \langle \xi(0) (Q b_{-1} Y \delta_i G) (Y \delta_j G) (\xi Y \delta_k G) \rangle + \\
& (-)^{1+\epsilon_i} \langle \xi(0) (b_{-1} Y \delta_i G) (Q Y \delta_j G) (\xi Y \delta_k G) \rangle + (-)^{1+\epsilon_i + \epsilon_j} \langle \xi(0) (b_{-1} Y \delta_i G) (Y \delta_j G) \delta_k G \rangle + \\
& (-)^{\epsilon_i + \epsilon_j} \langle \xi(0) (b_{-1} Y \delta_i G) (Y \delta_j G) (\xi (Q Y \delta_k G)) \rangle + \\
& (-)^{\epsilon_i + \epsilon_j + \epsilon_k} \langle \xi(0) (b_{-1} Y \delta_i G) (Y \delta_j G) (\xi Y \delta_k G) [Q, e^{\int d\tau d\theta \mathcal{V}}] \rangle_0 = \\
& \langle X(0) (b_{-1} Y \delta_i G) (Y \delta_j G) (\xi Y \delta_k G) \rangle.
\end{aligned}$$

We have explicitly inserted a ξ insertion in the center of the disk to saturate the zero mode of the $\xi\eta$ system [25]. This insertion is absent from similar expressions in the main text, but its presence is always implied. The left hand side of these Ward identities is obtained by pushing the BRST current contour onto the boundary and the right hand side by shrinking it to zero radius around the center of the disk.

Solving in the above identities for $\langle \xi(0) (b_{-1} Y \delta_i G) \delta_j G (Y \delta_k G) \rangle$ and $\langle \xi(0) (b_{-1} Y \delta_i G) (Y \delta_j G) \delta_k G \rangle$ and adding the resulting expressions gives

$$\begin{aligned}
& \langle \xi(0) (b_{-1} Y \delta_i G) \delta_j G (Y \delta_k G) \rangle + \langle \xi(0) (b_{-1} Y \delta_i G) (Y \delta_j G) \delta_k G \rangle = \\
& (-)^{\epsilon_i} \langle \xi(0) (Q b_{-1} Y \delta_i G) (\xi Y \delta_j G) (Y \delta_k G) \rangle + (-)^{\epsilon_i + \epsilon_j} \langle \xi(0) (Q b_{-1} Y \delta_i G) (Y \delta_j G) (\xi Y \delta_k G) \rangle + \\
& \langle \xi(0) (b_{-1} Y \delta_i G) (\xi (Q Y \delta_j G)) (Y \delta_k G) \rangle + (-)^{\epsilon_j + 1} \langle \xi(0) (b_{-1} Y \delta_i G) (Q Y \delta_j G) (\xi Y \delta_k G) \rangle + \\
& (-)^{\epsilon_i + 1} \langle X(0) (b_{-1} Y \delta_i G) (\xi Y \delta_j G) (Y \delta_k G) \rangle +
\end{aligned}$$

$$\begin{aligned}
& (-)^{\epsilon_j} \langle \xi(0)(b_{-1}Y\delta_i G)(\xi Y\delta_j G)(QY\delta_k G) \rangle + \langle \xi(0)(b_{-1}Y\delta_i G)(Y\delta_j G)(\xi(QY\delta_k G)) \rangle + \\
& (-)^{\epsilon_j+\epsilon_k} \langle \xi(0)(b_{-1}Y\delta_i G)(\xi Y\delta_j G)(Y\delta_k G)[Q, e^{\int d\tau d\theta \mathcal{V}}] \rangle_0 + \\
& (-)^{\epsilon_k} \langle \xi(0)(b_{-1}Y\delta_i G)(Y\delta_j G)(\xi Y\delta_k G)[Q, e^{\int d\tau d\theta \mathcal{V}}] \rangle_0 + \\
& (-)^{\epsilon_j+\epsilon_i+1} \langle X(0)(b_{-1}Y\delta_i G)(Y\delta_j G)(\xi Y\delta_k G) \rangle.
\end{aligned}$$

Since, the position of the ξ insertions is irrelevant, we can move appropriately the ξ insertion (on the boundary) in the above expressions and after integrating over τ and θ we take

$$\int \prod_{\beta=1}^3 d\tau_\beta d\theta_\beta \left(\langle (b_{-1}Y\delta_i G)\delta_j G Y\delta_k G \rangle + \langle (b_{-1}Y\delta_i G)Y\delta_j G \delta_k G \rangle \right) = 0. \quad (90)$$

Continuing in the same fashion for the other two pairs we conclude that $d\omega = 0$.

The proof of V -invariance of ω goes in a similar way. The only addition is the use of the identity $\{Q, b_{-1}\} = v^a \partial_a$, where $v^a \partial_a$ is the generator of rotations on the disk, as well as the use of the identity

$$\int d\tau d\theta v^a \partial_a (Y\delta G) = 0. \quad (91)$$

Appendix B

In this appendix we would like to show equation (42). From (41) we have

$$[Q, C\mathcal{V}] = [(1-h)C\partial_\tau C - \frac{1}{4}(D_\theta C)(D_\theta C)]\mathcal{V} + \frac{1}{2}C(D_\theta C)(D_\theta \mathcal{V}). \quad (92)$$

In components, the above equation involves the expressions ⁷

$$C\partial_\tau C = c\partial_\tau c + \theta(\gamma\partial_\tau c - c\partial_\tau \gamma), \quad (93)$$

$$(D_\theta C)(D_\theta C) = \gamma^2 + 2\theta\gamma\partial_\tau c, \quad (94)$$

$$CD_\theta C = c\gamma + \theta(\gamma^2 - c\partial_\tau c). \quad (95)$$

⁷ All products of fields appearing here are normal ordered in the usual CFT fashion, i.e. $AB(w) = \oint \frac{dz}{2\pi i} \frac{1}{z-w} A(z)B(w)$.

Since $Y = -\partial\xi ce^{-2\phi}$ we easily deduce by using the relevant OPEs that

$$Y\gamma = -ce^{-\phi}. \quad (96)$$

Hence, acting with Y on the above equations gives the following expressions

$$YC\partial_\tau C = Yc\partial_\tau c + \theta Y\gamma\partial_\tau c = Yc\partial_\tau c - \theta c\partial_\tau ce^{-\phi}, \quad (97)$$

$$Y(D_\theta C)(D_\theta C) = Y\gamma^2 + 2\theta Y\gamma\partial_\tau c = Y\gamma^2 - 2\theta c\partial_\tau ce^{-\phi}, \quad (98)$$

$$YCD_\theta C = \theta(Y\gamma^2 - Yc\partial_\tau c). \quad (99)$$

Combining these equations with (92) gives

$$\begin{aligned} Y[Q, C\mathcal{V}] &= \left((1-h)(Yc\partial_\tau c - \theta c\partial_\tau ce^{-\phi} - \frac{1}{4}Y\gamma^2 + 2\frac{1}{4}\theta c\partial_\tau ce^{-\phi}) \right) \mathcal{V} + \frac{1}{2}\theta(Y\gamma^2 - Yc\partial_\tau c)D_\theta \mathcal{V} = \\ &= \left((1-h)Yc\partial_\tau c - \frac{1}{4}Y\gamma^2 \right) \mathcal{V} + \frac{1}{2}\theta(Y\gamma^2 - Yc\partial_\tau c)D_\theta \mathcal{V} + \left(h - \frac{1}{2} \right) \theta c\partial_\tau ce^{-\phi} \mathcal{V} = \\ &= \left((1-h)Yc\partial_\tau c - \frac{1}{4}Y\gamma^2 \right) \mathcal{V} + \frac{1}{2}\theta(Y\gamma^2 - Yc\partial_\tau c)D_\theta \mathcal{V} + \left(h - \frac{1}{2} \right) \theta c\partial_\tau ce^{-\phi} D, \end{aligned} \quad (100)$$

i.e. equation (42).

Appendix C

In this appendix we use the SUSY Ward identities on the real line in order to demonstrate relations between the coefficients of certain 2-point and 3-point functions at the conformal point.

2-point functions

The SUSY Ward identity for the 2-point functions on the real line reads

$$\begin{aligned} \delta\langle U_i(x_1)D_j(x_2)\rangle_0 &= 0 \Rightarrow \\ \langle \partial_{x_1} D_i(x_1)D_j(x_2)\rangle_0 + \langle U_i(x_1)U_j(x_2)\rangle_0 &= 0, \end{aligned} \quad (101)$$

where δ denotes an infinitesimal SUSY transformation.

Plugging in (101) the CFT expressions

$$\langle D_i(x_1)D_j(x_2)\rangle_0 = \frac{c\delta_{h_i, h_j}}{|x_1 - x_2|^{2h}} \text{sign}(x_1 - x_2) \quad (102)$$

and

$$\langle U_i(x_1)U_j(x_2) \rangle_0 = \frac{c\delta_{h_i, h_j}}{|x_1 - x_2|^{2h+1}} \quad (103)$$

with $h = h_i = h_j$, we conclude that

$$2ch = b. \quad (104)$$

3-point functions

The relevant Ward identity is

$$\begin{aligned} & \delta \langle U_i(x_1)D_j(x_2)U_k(x_3) \rangle_0 \Rightarrow \\ & \langle \partial_{x_1} D_i(x_1)D_j(x_2)U_k(x_3) \rangle_0 + \langle U_i(x_1)U_j(x_2)U_k(x_3) \rangle_0 - \langle U_i(x_1)D_j(x_2)\partial_{x_3} D_k(x_3) \rangle_0 = 0. \end{aligned} \quad (105)$$

Substituting for the CFT expressions

$$\langle U_i(x_1)U_j(x_2)U_k(x_3) \rangle_0 = \frac{B_{ijk}}{|x_1 - x_2|^{h_i+h_j-h_k+\frac{1}{2}}|x_2 - x_3|^{h_k+h_j-h_i+\frac{1}{2}}|x_3 - x_1|^{h_i+h_k-h_j+\frac{1}{2}}}, \quad (106)$$

$$\langle D_i(x_1)D_j(x_2)U_k(x_3) \rangle_0 = \frac{C_{ijk} \text{ sign}(x_1 - x_2)}{|x_1 - x_2|^{h_i+h_j-h_k-\frac{1}{2}}|x_2 - x_3|^{h_j+h_k-h_i+\frac{1}{2}}|x_3 - x_1|^{h_i+h_k-h_j+\frac{1}{2}}} \quad (107)$$

gives

$$\begin{aligned} B_{ijk}(x_1 - x_3) &= C_{ijk} \left(h_i + h_j - h_k - \frac{1}{2} \right) (x_1 - x_3) + C_{ijk} \left(h_k + h_i - h_j + \frac{1}{2} \right) (x_1 - x_2) + \\ &+ C_{jki} \left(h_j + h_k - h_i - \frac{1}{2} \right) (x_1 - x_3) - C_{jki} \left(h_i + h_k - h_j + \frac{1}{2} \right) (x_3 - x_2). \end{aligned} \quad (108)$$

This equation must be valid for any value of the worldsheet variables x_1, x_2, x_3 . Hence,

$$B_{ijk} = 2h_i C_{ijk} + C_{jki} \left(h_k + h_j - h_i - \frac{1}{2} \right), \quad (109)$$

$$B_{ijk} = C_{ijk}(h_i + h_j - h_k) + 2h_k C_{jki}, \quad (110)$$

$$C_{ijk} = C_{jki}. \quad (111)$$

Equivalently, the first two equations give

$$B_{ijk} = -C_{ijk} \left(\frac{1}{2} - h_i - h_j - h_k \right). \quad (112)$$

References

- [1] E. Witten, “On Background Independent Open-String Field Theory,” *Phys. Rev.* **D47** (1992) 5467; hep-th/9208027.
- [2] E. Witten, “Some Computations in Background Independent Open-String Field Theory,” *Phys. Rev.* **D47** (1993) 3405; hep-th/9210065.
- [3] S. Shatashvili, “Comment on the Background Independent Open String Theory,” *Phys. Lett.* **B311** (1993) 83; hep-th/9303143.
- [4] S. Shatashvili, “On the Problems with Background Independence in String Theory,” hep-th/9311177.
- [5] A. Gerasimov, S. Shatashvili, “On Exact Tachyon Potential in Open String Field Theory,” *JHEP* **0010** (2000) 034; hep-th/0009103.
- [6] D. Kutasov, M. Marino, G. Moore, “Some Exact Results on Tachyon Condensation in String Field Theory,” *JHEP* **0010** (2000) 045; hep-th/0009148.
- [7] D. Kutasov, M. Marino, G. Moore, “Remarks on Tachyon Condensation in Superstring Field Theory,” hep-th/0010108.
- [8] J. Harvey, D. Kutasov, E. Martinec, “On the Relevance of Tachyons,” hep-th/0003101.
- [9] A. Sen, “Descent Relations Among Bosonic D-branes,” *Int. J. Mod. Phys.* **A14** (1999) 4061.
- [10] I. Batalin, G. Vilkovisky, “Quantization of Gauge Systems with Linearly Dependent Generators,” *Phys. Rev.* **D28** (1983) 2567.
- [11] I. Batalin, G. Vilkovisky, “Existence Theorem for Gauge Algebras,” *J. Math. Phys.* **26** (1985) 172.
- [12] O. Andreev, A. Tseytlin, “Partition Function Representation for the Open Superstring Action,” *Nucl. Phys.* **B311** (1988) 205.
- [13] A. Tseytlin, “Sigma Model Approach to String Theory,” *Int. Jour. Mod. Phys.* **A4** (1989) 1257.
- [14] A. Tseytlin, “Renormalization Group and String Loops,” *Int. Jour. Mod. Phys.* **A5** (1990) 589.
- [15] A.A. Tseytlin, “Sigma Model Approach to String Theory Effective Actions with Tachyons,” hep-th/0011033.
- [16] P. Kraus, F. Larsen, “Boundary String Field Theory of the DDbar System,” hep-th/0012198.
- [17] T. Takayanagi, S. Terashima, T. Uesugi, “Brane-Antibrane Action from Boundary String Field Theory,” hep-th/0012210.
- [18] M. Marino, “On the BV Formulation of Boundary Superstring Field Theory,” hep-th/0103089.
- [19] M. Bochicchio, “Gauge Fixing for the Field Theory of the Bosonic String,” *Phys. Lett.* **B193** (1987) 31.

- [20] C. Thorn, “String Field Theory,” *Phys. Rep.* **174** (1989) 1.
- [21] B. Zwiebach, “Closed String Field Theory: Quantum Action and the Batalin-Vilkovisky Master Equation,” *Nucl. Phys.* **B390** (1993) 33; hep-th/9206084.
- [22] J. Gomis, J. Paris, S. Samuel, “Antibracket, Antifields and Gauge Theory Quantization,” hep-th/9412228.
- [23] M. Henneaux, C. Teitelboim, “Quantization of Gauge Systems,” Princeton University Press (1992).
- [24] B. Zwiebach, “Closed String Field Theory: An Introduction,” hep-th/9305026.
- [25] J. Polchinski, “String Theory,” Vol. 2, Cambridge University Press (1998).
- [26] D. Friedan, E. Martinec, S. Shenker, “Conformal Invariance, Supersymmetry, and String Theory,” *Nucl. Phys.* **B27** (1986) 93.
- [27] E. Witten, “Interacting Field Theory of Open Superstrings,” *Nucl. Phys.* **B276** (1986) 291.
- [28] A. Belopolsky, “Picture Changing Operators in Supergeometry and Superstring Theory,” hep-th/9706033.
- [29] C. Preitschopf, C. Thorn, S. Yost, “Superstring Field Theory,” *Nucl. Phys.* **B337** (1990) 363.
- [30] I. Ya. Aref’eva, P. Medvedev, A. Zubarev, “Background Formalism for Superstring Field Theory,” *Phys. Lett.* **240B** (1990) 356.
- [31] I. Ya. Aref’eva, P. Medvedev, A. Zubarev, “New Representation for String Field Solves the Consistency Problem for Open Superstring Field Theory,” *Nucl. Phys.* **B341** (1990) 464.
- [32] I. Affleck, W. Ludwig, “Universal Noninteger “Ground State Degeneracy” in Critical Quantum Systems,” *Phys. Rev. Lett.* **67** (1991) 161.
- [33] I. Affleck, W. Ludwig, “Exact Conformal Field Theory Results on the Multichannel Kondo Effect: Single Fermion Green’s Function, Self-energy, and Resistivity,” *Phys. Rev.* **B48** (1993) 7297.
- [34] K. Li, E. Witten, “Role of Short Distance Behavior in Off-Shell Open-String Field Theory,” *Phys. Rev.* **D48** (1993) 853; hep-th/9303067.